Last time: Problem-Solving

- **Problem solving:**
  - Goal formulation
  - Problem formulation (states, operators)
  - Search for solution

- **Problem formulation:**
  - Initial state
  - ?
  - ?
  - ?

- **Problem types:**
  - single state: accessible and deterministic environment
  - multiple state: ?
  - contingency: ?
  - exploration: ?

Last time: Problem-Solving

- **Problem solving:**
  - Goal formulation
  - Problem formulation (states, operators)
  - Search for solution

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  - Initial state
  - Operators
  - Goal test
  - Path cost

- **Problem types:**
  - single state: accessible and deterministic environment
  - multiple state: ?
  - contingency: ?
  - exploration: ?

Last time: Problem-Solving

- **Problem solving:**
  - Goal formulation
  - Problem formulation (states, operators)
  - Search for solution

- **Problem formulation:**
  - Initial state
  - Operators
  - Goal test
  - Path cost

- **Problem types:**
  - single state: accessible and deterministic environment
  - multiple state: inaccessible and deterministic environment
  - contingency: inaccessible and nondeterministic environment
  - exploration: unknown state-space

Last time: Finding a solution

Solution: is ???

Basic idea: offline, systematic exploration of simulated state-space by generating successors of explored states (expanding)

```
Function General-Search(problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add resulting nodes to the search tree
    end
```

Last time: Finding a solution

Solution: is a sequence of operators that bring you from current state to the goal state.

Basic idea: offline, systematic exploration of simulated state-space by generating successors of explored states (expanding).

Function General-Search(*problem*, *strategy*) returns a *solution*, or failure
    initialize the search tree using the initial state problem
    loop do
        if there are no candidates for expansion **then return** failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state **then return** the corresponding solution
        else expand the node and add resulting nodes to the search tree
    end

Strategy: The search strategy is determined by ???

Last time: Finding a solution

Solution: is a sequence of operators that bring you from current state to the goal state

Basic idea: offline, systematic exploration of simulated state-space by generating successors of explored states (expanding)

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        if the node contains a goal state then return the corresponding solution
        else expand the node and add resulting nodes to the search tree
    end
```

Strategy: The search strategy is determined by the **order** in which the nodes are expanded.

A Clean Robust Algorithm

```
Function UniformCost-Search(problem, Queuing-Fn) returns a solution, or failure
    open ← make-queue(make-node(initial-state[problem]))
    closed ← [empty]
    loop do
        if open is empty then return failure
        currnode ← Remove-Front(open)
        if Goal-Test[problem] applied to State(currnode) then return currnode
        children ← Expand(currnode, Operators[problem])
        while children not empty

                        [… see next slide …]
        end
        closed ← Insert(closed, currnode)
        open ← Sort-By-PathCost(open)
    end
```

A Clean Robust Algorithm

```
[... see previous slide ...]
      children ← Expand(currnode, Operators[problem])
      while children not empty
            child ← Remove-Front(children)
            if no node in open or closed has child's state
                  open ← Queuing-Fn(open, child)
            else if there exists node in open that has child's state
                  if PathCost(child) < PathCost(node)
                        open ← Delete-Node(open, node)
                        open ← Queuing-Fn(open, child)
            else if there exists node in closed that has child's state
                  if PathCost(child) < PathCost(node)
                        closed ← Delete-Node(closed, node)
                        open ← Queuing-Fn(open, child)
      end
[... see previous slide ...]
```

Last time: search strategies

Uninformed: Use only information available in the problem formulation

- Breadth-first
- Uniform-cost
- Depth-first
- Depth-limited
- Iterative deepening

Informed: Use heuristics to guide the search

- Best first
- A*

Evaluation of search strategies

- Search algorithms are commonly evaluated according to the following four criteria:
  - **Completeness:** does it always find a solution if one exists?
  - **Time complexity:** how long does it take as a function of number of nodes?
  - **Space complexity:** how much memory does it require?
  - **Optimality:** does it guarantee the least-cost solution?

- Time and space complexity are measured in terms of:
  - b – max branching factor of the search tree
  - d – depth of the least-cost solution
  - m – max depth of the search tree (may be infinity)

Last time: uninformed search strategies

Uninformed search:

Use only information available in the problem formulation

- Breadth-first
- Uniform-cost
- Depth-first
- Depth-limited
- Iterative deepening

This time: informed search

Informed search:

Use heuristics to guide the search

- Best first
- A*
- Heuristics
- Hill-climbing
- Simulated annealing

Best-first search

- Idea:

  use an evaluation function for each node; estimate of ***"desirability"***

$\Rightarrow$ expand most desirable unexpanded node.

- Implementation:

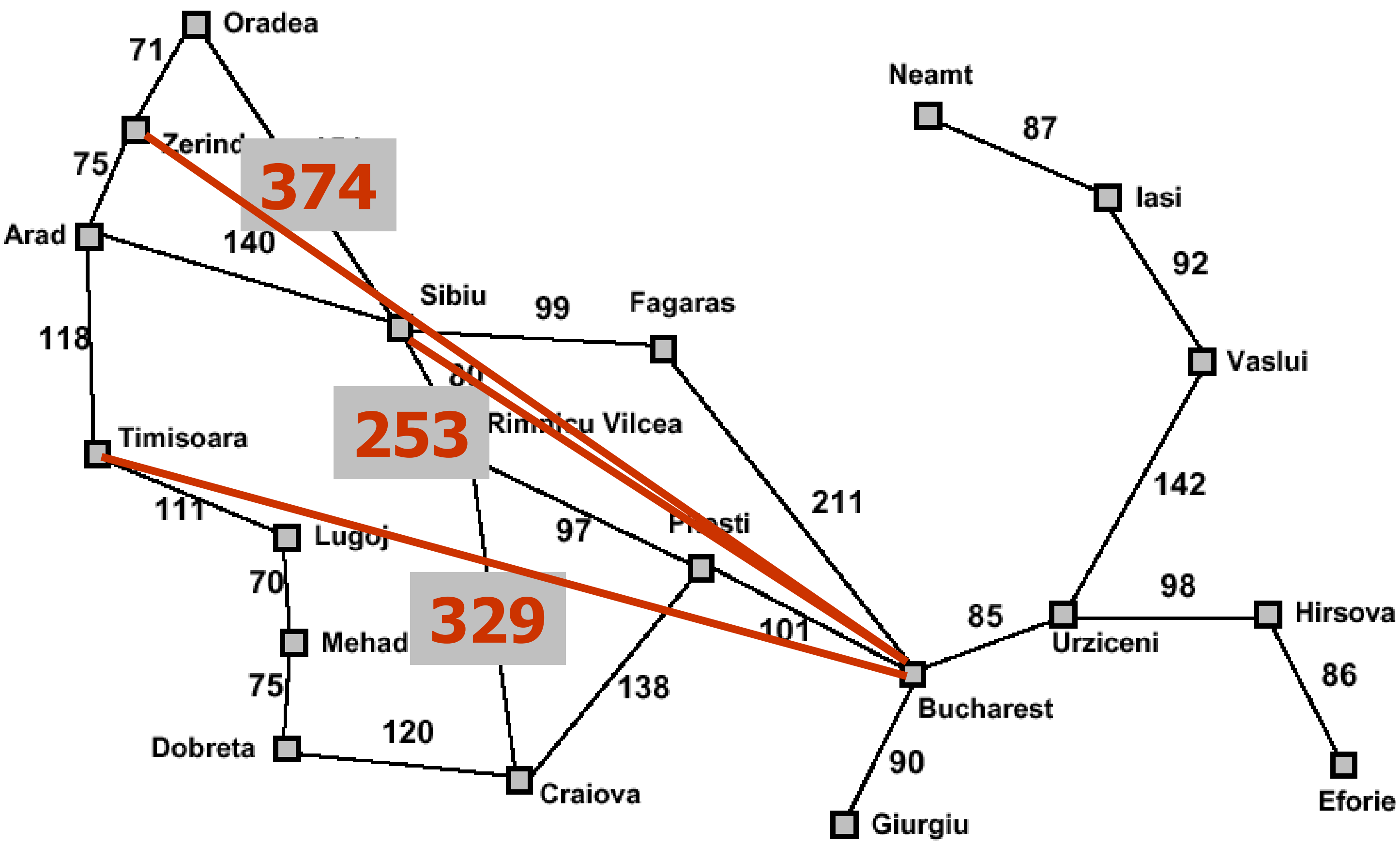
  QueueingFn = insert successors in decreasing order of desirability

- Special cases:

  greedy search

  A* search

Romania with step costs in km



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy search

- Estimation function:

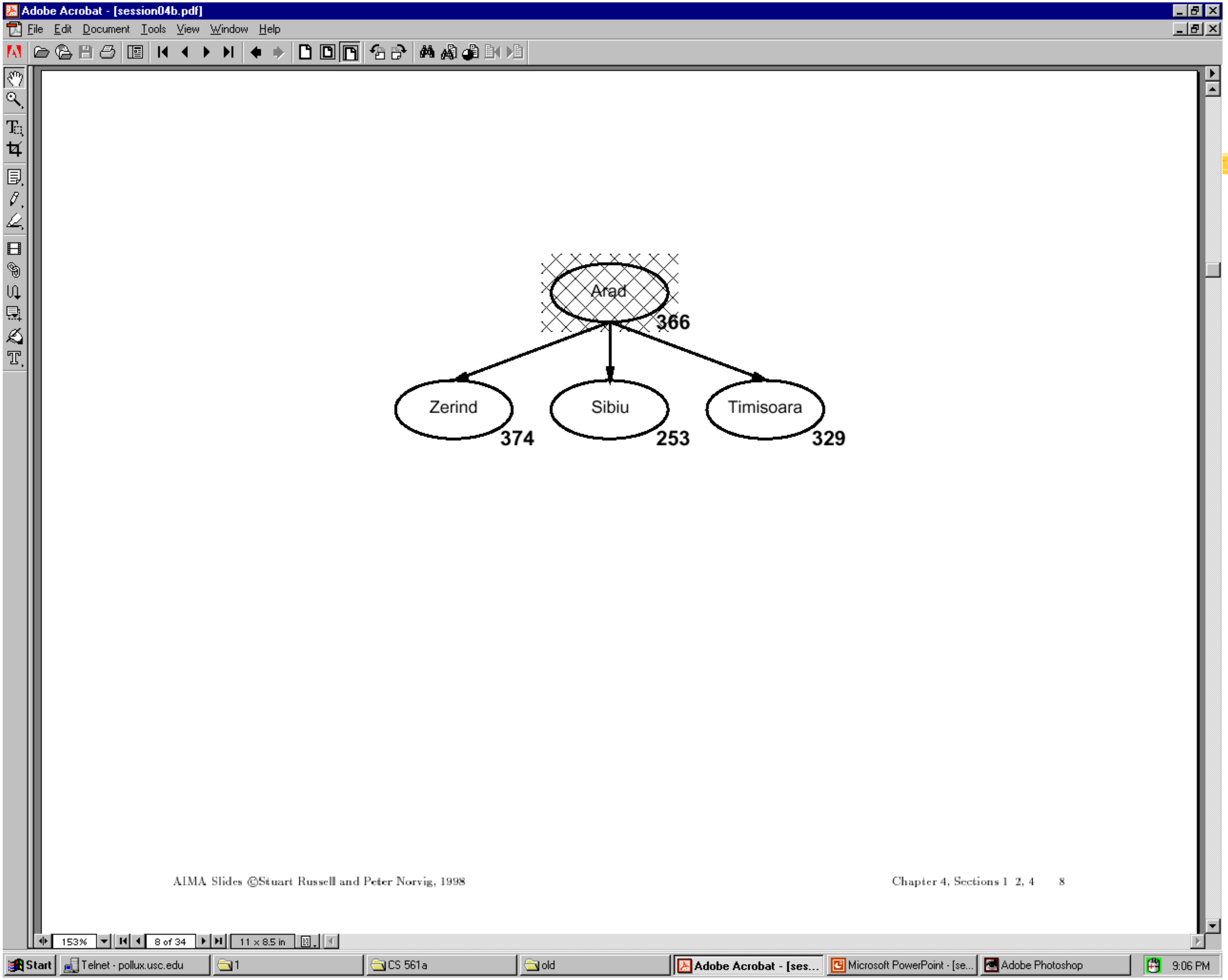
  $h(n)$ = estimate of cost from n to goal (heuristic)

- For example:

  $h_{SLD}(n)$ = straight-line distance from n to Bucharest

- Greedy search expands first the node that appears to be closest to the goal, according to $h(n)$.

Greedy search example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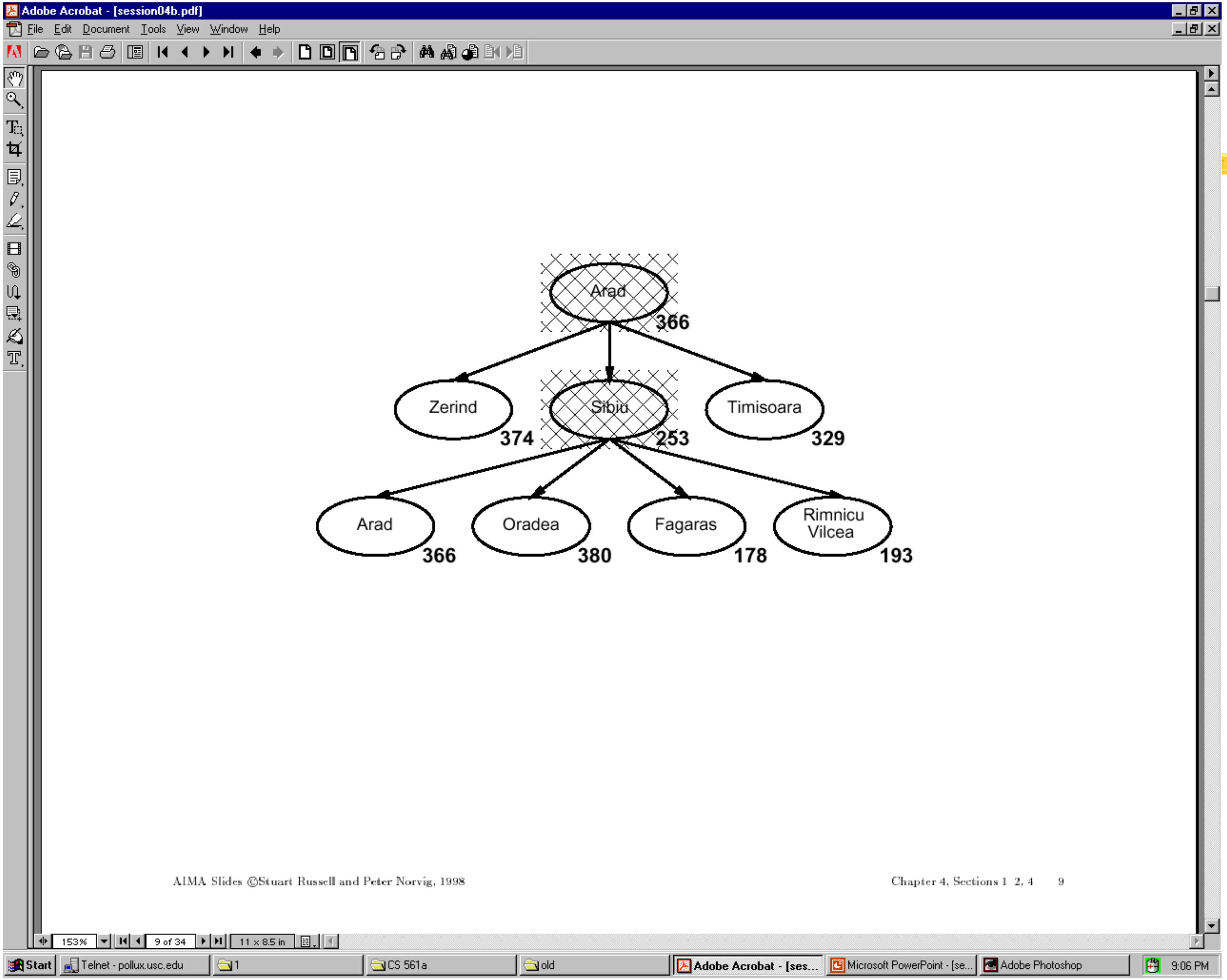
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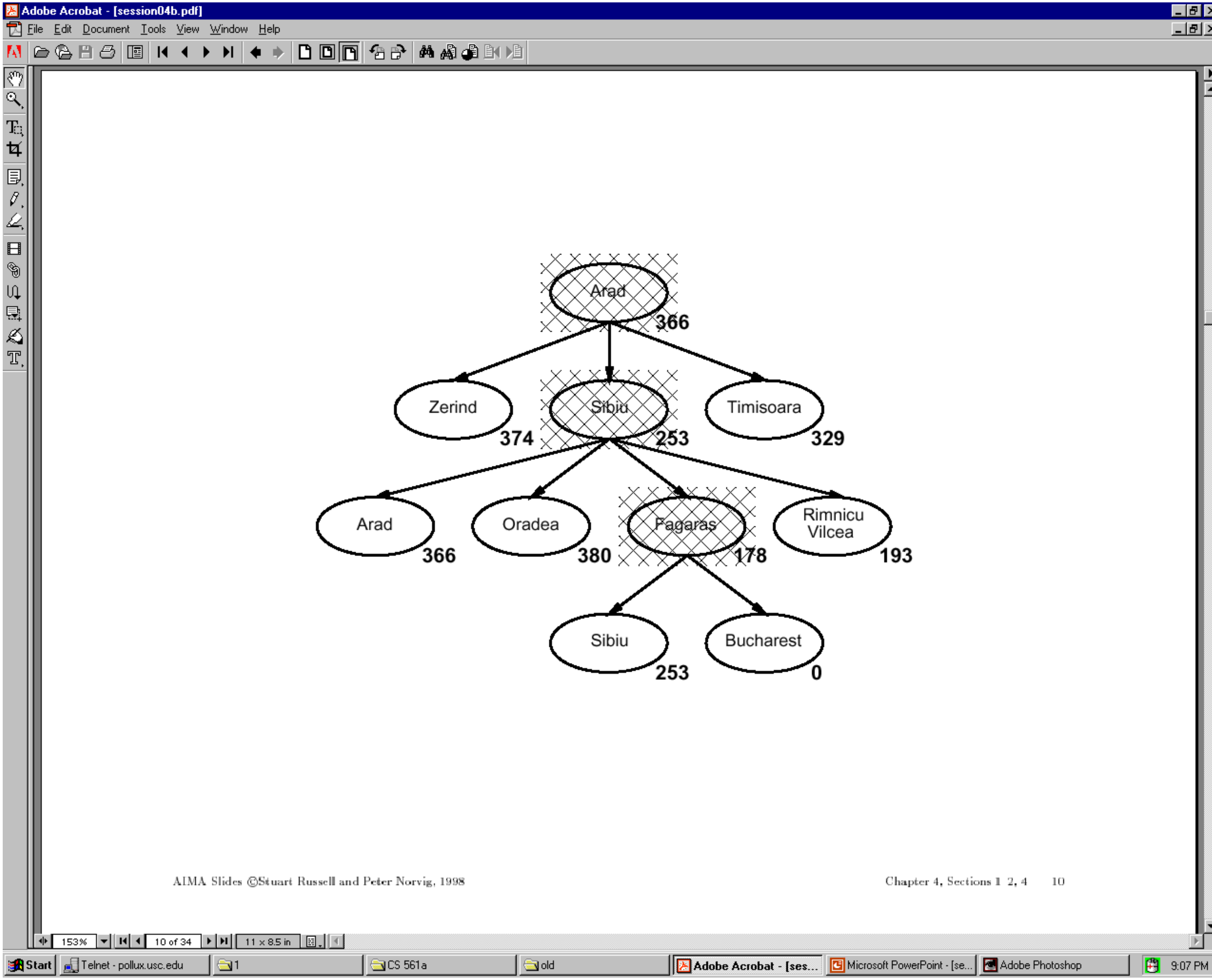
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Properties of Greedy Search

- Complete?
- Time?
- Space?
- Optimal?

Properties of Greedy Search

- Complete? No – can get stuck in loops
  e.g., Iasi > Neamt > Iasi > Neamt > …
  Complete in finite space with repeated-state checking.
- Time? O(b^m) but a good heuristic can give dramatic improvement
- Space? O(b^m) – keeps all nodes in memory
- Optimal? No.

A* search

- Idea: avoid expanding paths that are already expensive

  evaluation function: $f(n) = g(n) + h(n)$ with:

  $g(n)$ – cost so far to reach n

  $h(n)$ – estimated cost to goal from n

  $f(n)$ – estimated total cost of path through n to goal

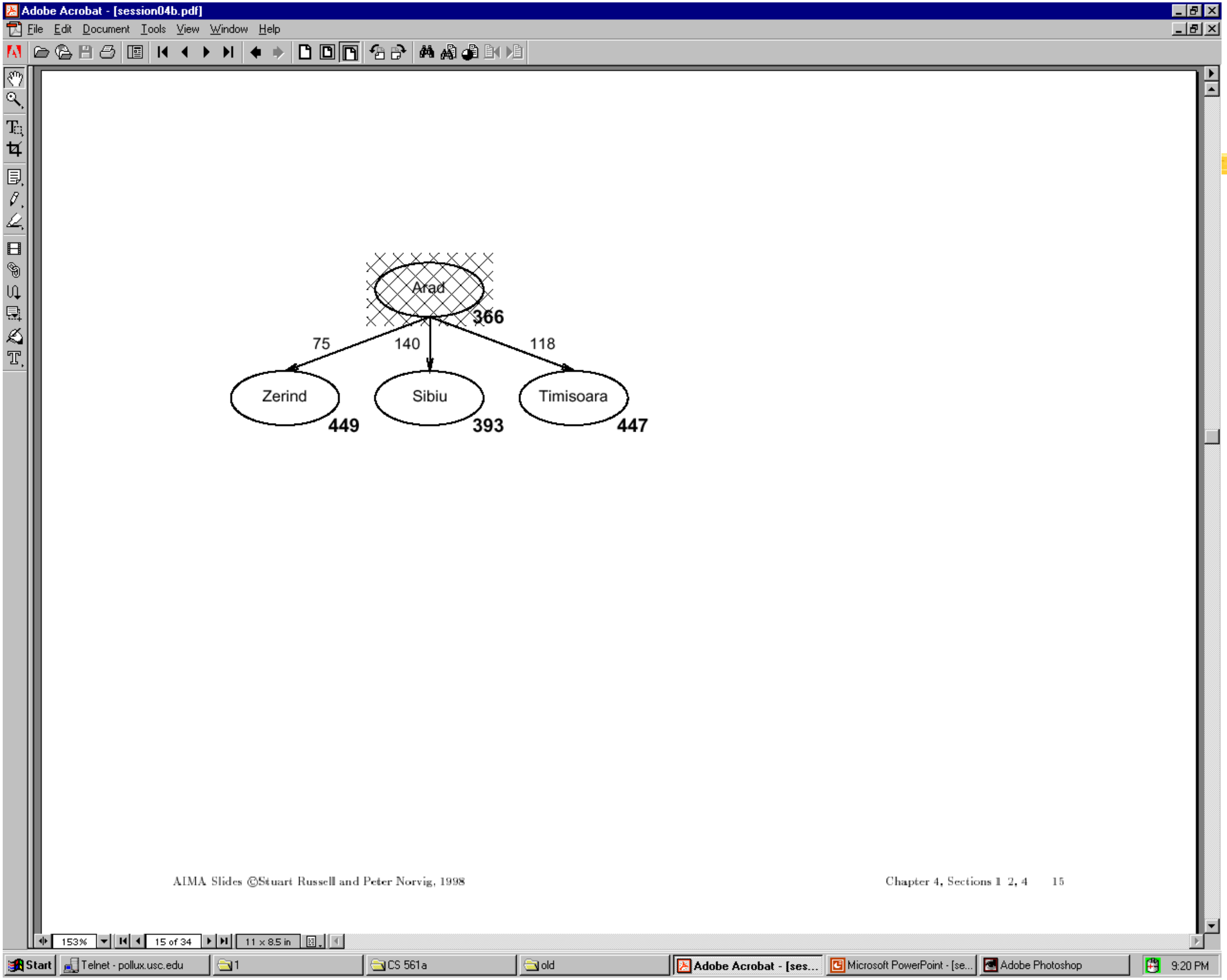
- A* search uses an admissible heuristic, that is,

  $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from n.

  For example: $h_{SLD}(n)$ never overestimates actual road distance.

- Theorem: A* search is optimal

A^* search example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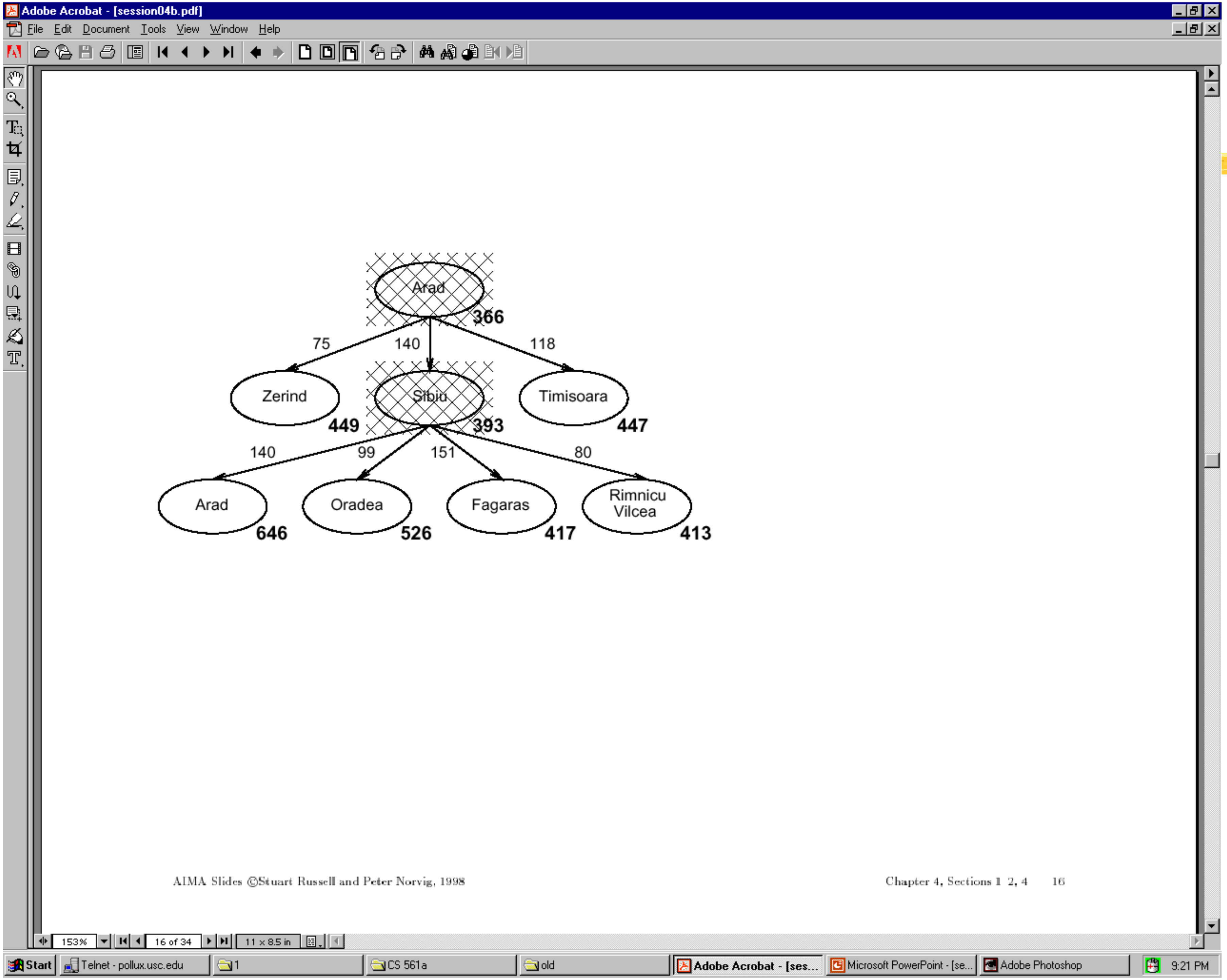
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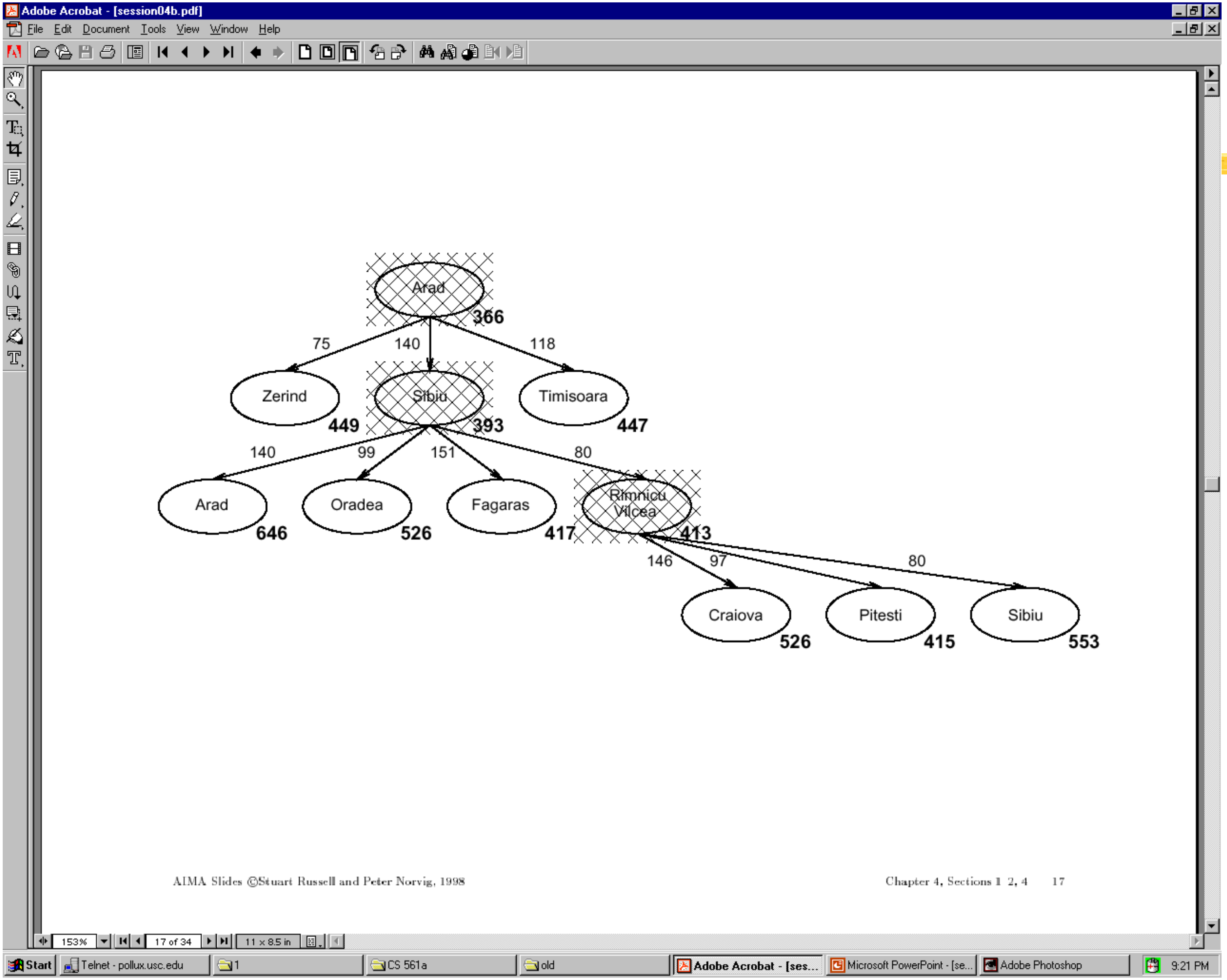
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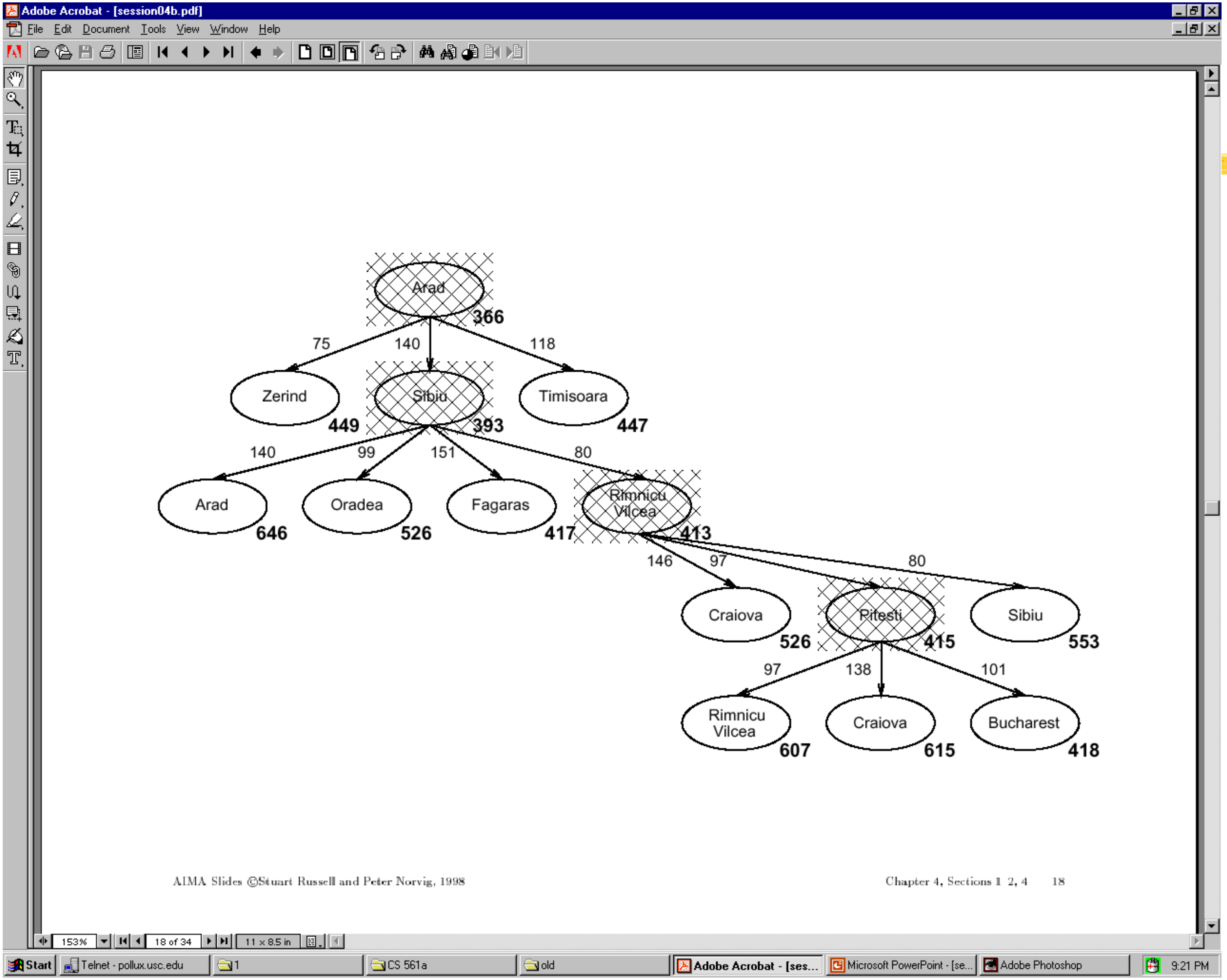
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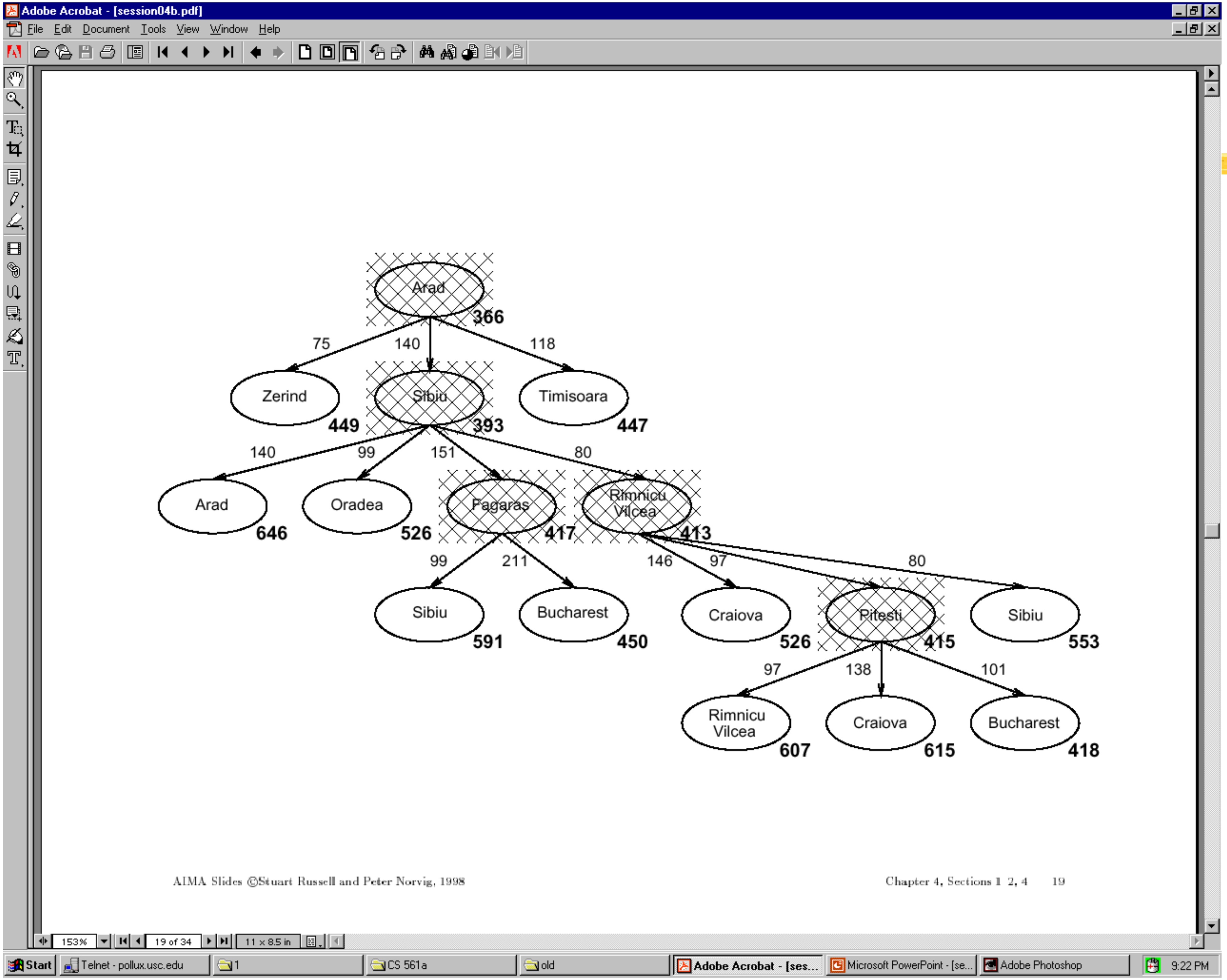
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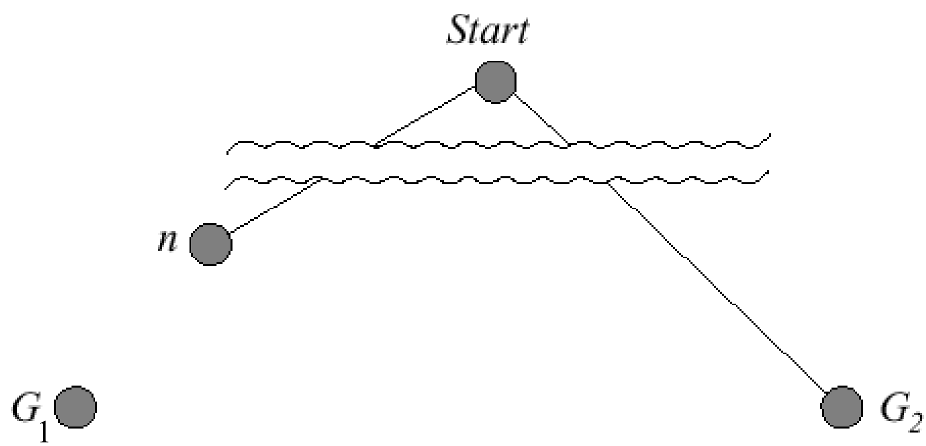
Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.



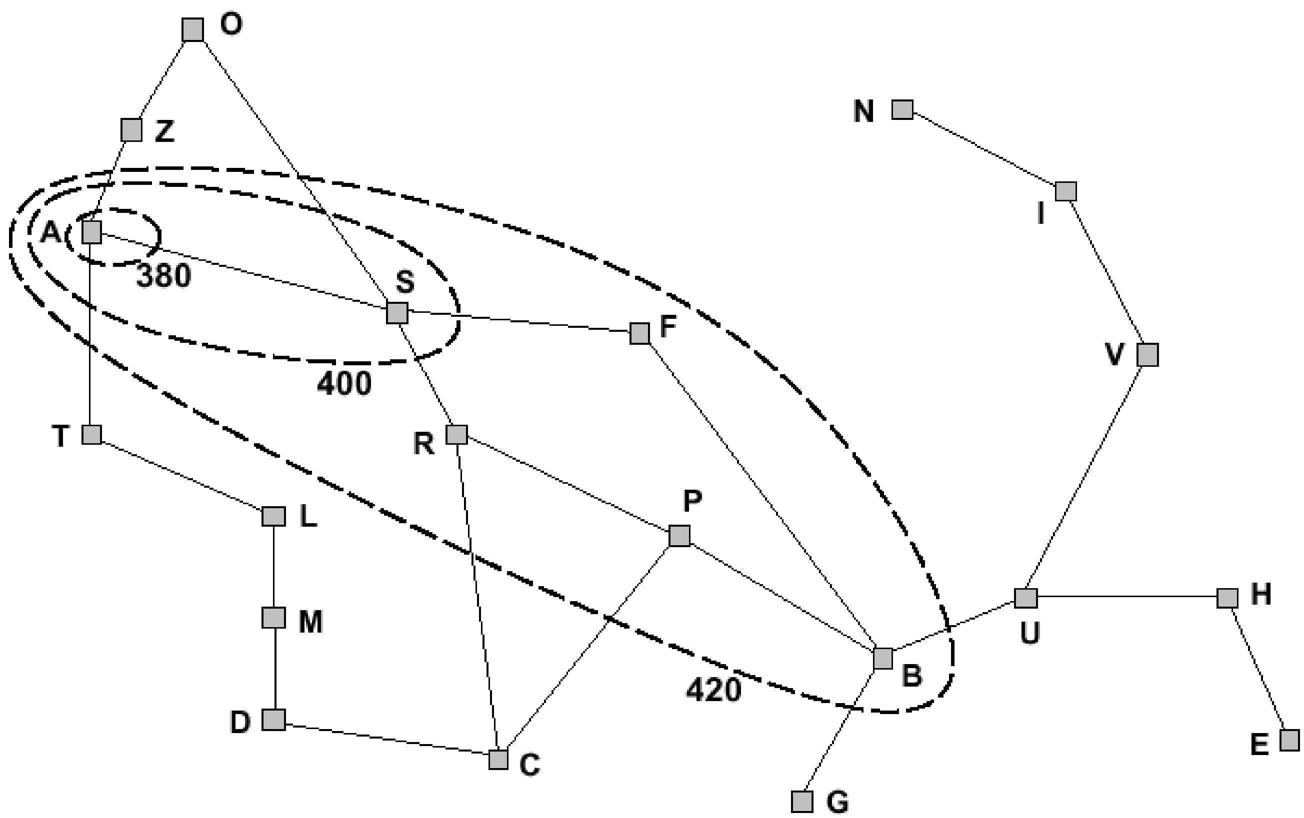
$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\ &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful proof)

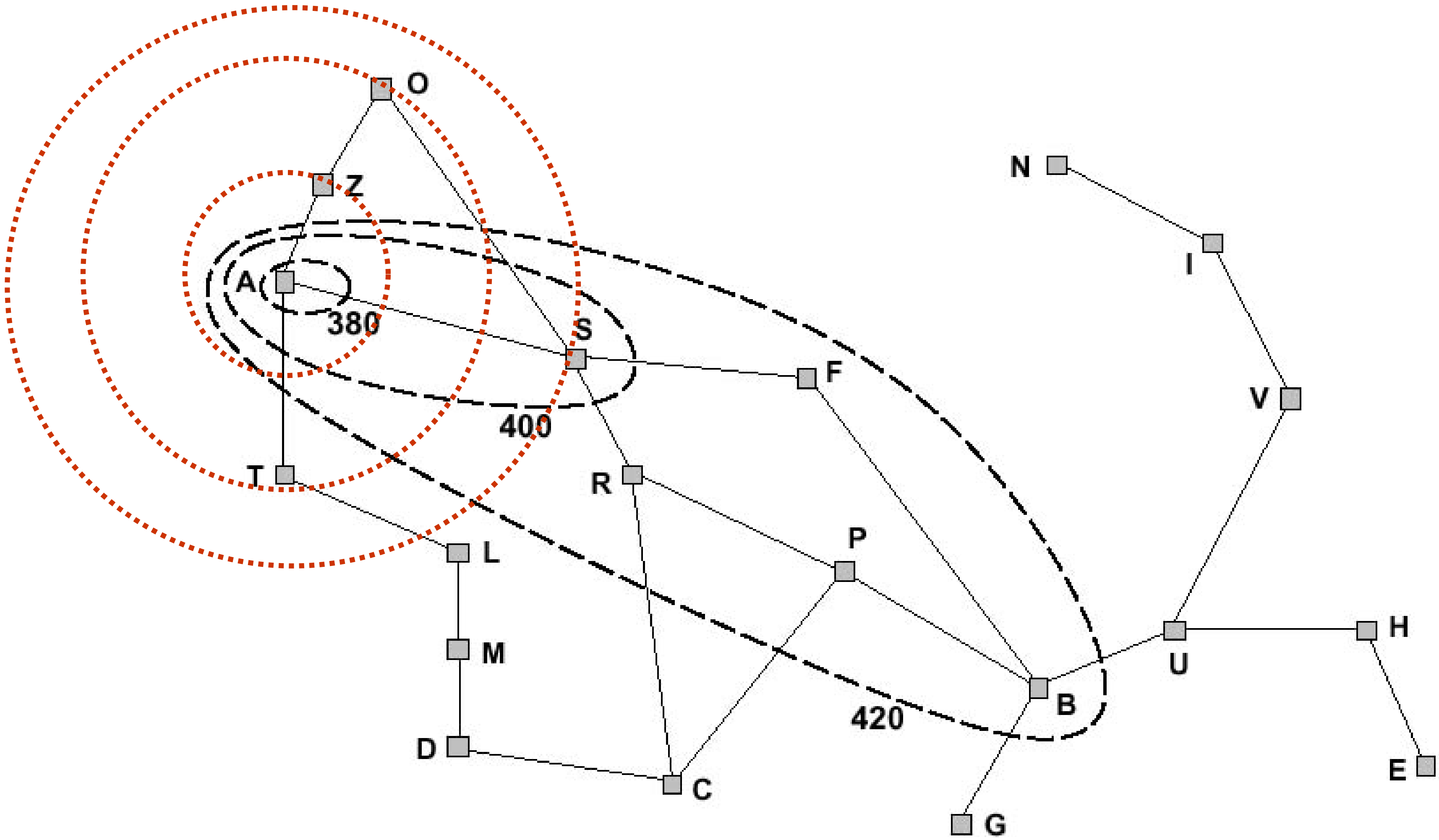
Lemma: A* expands nodes in order of increasing f value

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



f-contours

How do the contours look like when h(n) =0?



Properties of A*

- Complete?
- Time?
- Space?
- Optimal?

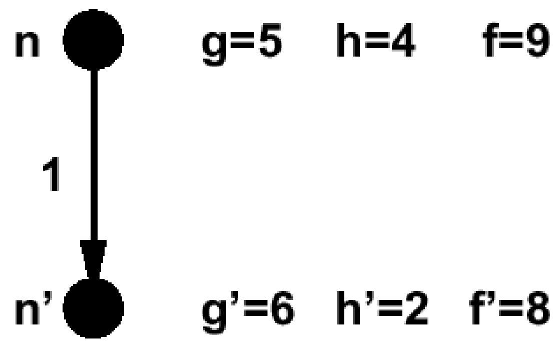
Properties of A*

- Complete? Yes, unless infinitely many nodes with $f \leq f(G)$
- Time? Exponential in [(relative error in h) x (length of solution)]
- Space? Keeps all nodes in memory
- Optimal? Yes – cannot expand f_{i+1} until f_i is finished

Proof of lemma: pathmax

For some admissible heuristics, f may *decrease* along a path

E.g., suppose n' is a successor of n



But this throws away information!
$f(n) = 9 \Rightarrow$ true cost of a path through n is ≥ 9
Hence true cost of a path through n' is ≥ 9 also

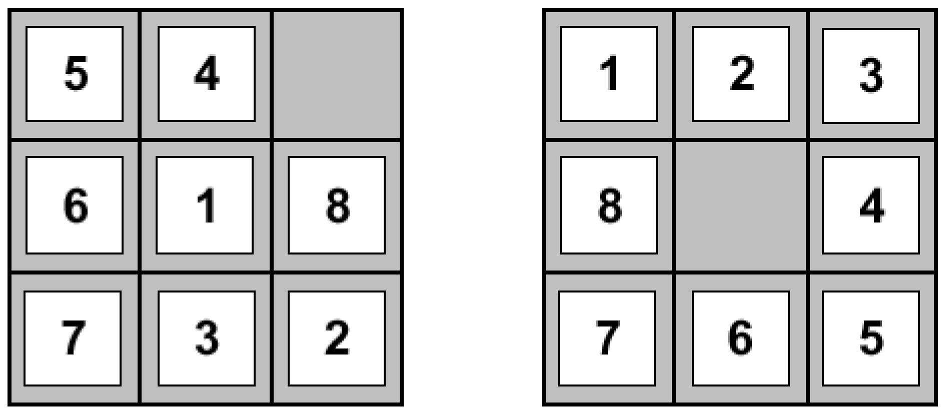
Pathmax modification to A*:
Instead of $f(n') = g(n') + h(n')$, use $f(n') = max(g(n') + h(n'), f(n))$

With pathmax, f is always nondecreasing along any path

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



Start State Goal State

$h_1(S) = ??$
$h_2(S) = ??$

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)

5	4	
6	1	8
7	3	2

Start State

1	2	3
8		4
7	6	5

Goal State

$h_1(S) = ??$ 7
$h_2(S) = ??$ 2+3+3+2+4+2+0+2 = 18

Relaxed Problem

- Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem.

- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution.

- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution.

Next time

- Iterative improvement
- Hill climbing
- Simulated annealing